DETERMINATION OF THE CHARACTERISTIC IMPEDANCE OF SINGLE AND COUPLED LINES IN LAYERED DIELECTRIC MEDIA

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ABSTRACT

A method is presented by which the characteristic impedances of lines in layered dielectric media can be calculated in a new way. Lines of semi-infinite extend (e.g. microstrip lines or a coupled line system) which are connected by picewise sinusodial basisfunctions are fed by Δ - gap sources. Spectral domain analysis is used to solve the eigenvalue problem of the lines and to determine the transverse current distribution. The characteristic impedance is defined as the quotient of voltage of the Δ - gap source and the total current on the lines which is calculated using the method of moments. Thus, neither the voltage between a line and the ground nor the power transported by the coupled line system is used. Results for single lines, asymmetric coupled lines and coplanar lines are given.

INTRODUCTION

Methods for the determination of the characteristic impedances of simple microstrip lines can be divided into two groups: At lower frequencies one can obtain satisfying solutions by the use of a static analysis. At higher frequencies the dispersive behaviour of the line cannot be neglected. In this frequency range only a dynamic analysis gives correct results.

A definition of the characteristic impedance which includes the voltage between the line and the ground is unfit especially for higher frequencies. At these frequencies the electromagnetic problem cannot be described by a *TEM* analysis and the voltage depends on the path of integration between line and ground. In [1] the characteristic impedance for a single line in multilayered media is calculated using this method. For coupled lines embedded in multilayered dielectric media with or without a ground, this definition is unsuitable.

The characteristic impedance is mostly defined by a quotient of the transported power and the line current. For this definition the power density in all layers has to be integrated. Thus, we would have to know the electric and magnetic field at any point in a sectional area transverse to the line. This is very complicated for many layers.

This paper presents a method which avoids the above mentioned difficulties defining the characteristic impedance of lines in layered dielectric media in a new way.

METHOD

Consider a Δ-gap source U which is connected at each side with a line of semi-infinite extend (Fig. 1). The line is embedded in a layered dielectric medium where the n'th or $\hat{m}$'th layer represents a reflector or a dielectric half space.

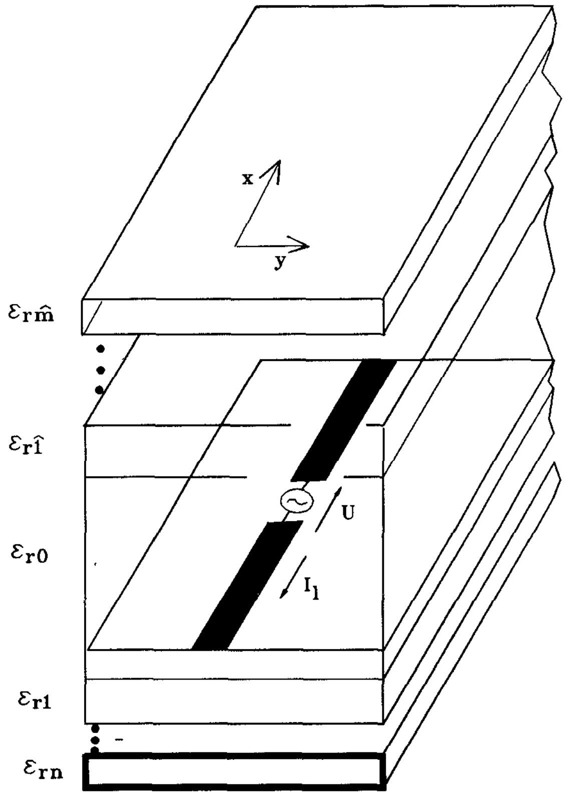

Fig. 1 Microstrip line fed by a Δ - gap source in layered dielectric media

The equivalent circuit (fig. 2) of this configuration gives directly an expression for the characteristic impedance:

$$Z_l = \left| \frac{U}{2I_l} \right| \quad . \tag{1}$$

Note, that in this definition the voltage U is directed longitudinally to the line and therefore exactly definable even if the circuit can not be described by *TEM* analysis.

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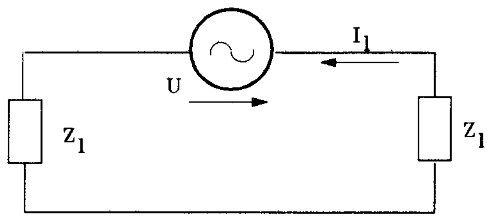

Fig. 2 Equivalent circuit of Fig. 1

For coupled lines we start in a similar way: This shall be illustrated using two examples. First consider two coupled lines (Fig. 3; for the layered dielectric media including a ground plane see Fig. 1). The even (c) and odd (π) modes each can be excited by two Δ–gap sources.

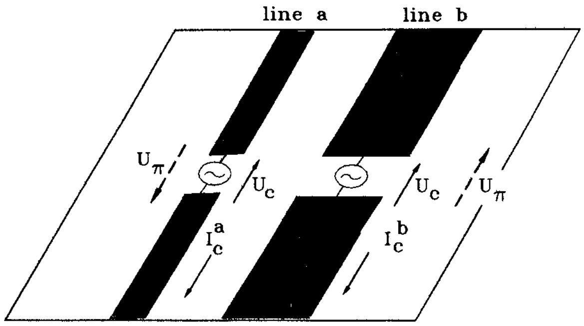

Fig. 3 Model for coupled lines (for the ground plane and the layered media see Fig. 1)

Then we get the 4 impedances:

$$Z_\pi^a = \left|\frac{U_\pi}{2I_\pi^a}\right| \qquad Z_\pi^b = \left|\frac{U_\pi}{2I_\pi^b}\right| \tag{2a}$$

$$Z_c^a = \left|\frac{U_c}{2I_c^a}\right| \qquad Z_c^b = \left|\frac{U_c}{2I_c^b}\right| . \tag{2b}$$

Also for a coplanar line (Fig. 4) the characteristic impedance can be determined easily. For the typically used mode we have to insert one Δ–gap source into the inner line and sources with an opposite direction in each ground line. Note that again the lines are embedded in layered dielectric media with or without a ground plane. For the characteristic impedance of a coplanar line we get

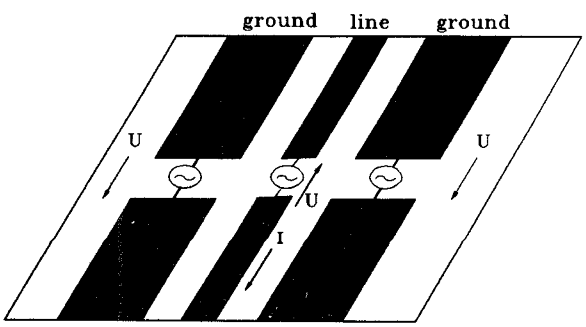

Fig. 4 Model for coplanar lines (for layered dielectric media see Fig. 1)

$$Z_{copl} = \left|\frac{U}{I}\right| \quad . \tag{3}$$

Corresponding models can be established in a similar way for other line geometries.

THEORETICAL BACKGROUND

The theory is based on full wave analysis ([2]). Therefore, first the Green's function G of the stratified medium is determined ([3],[4]). This Green's function connects a current density J in the plane $z = 0$ with its electric field

$$\vec{E}(\vec{r}) = \iint \overleftrightarrow{G}(\vec{r},\vec{r}\,')\vec{J}(\vec{r}\,')dxdy \quad . \tag{4}$$

The propagation constant k_x^{line} of the coupled line system and the transverse current distribution $f_k(y)$ of each of the K lines are calculated solving an eigenvalue problem. Therefore the lines are divided into N small strips with amplitudes A_n (Fig. 5). Thus the current density on the lines reads

$$\vec{J}^{lines}(x,y) = I_0 \sum_n^N \frac{A_n}{w_{yn}} \text{rect}\left(\frac{y-y_n}{w_{yn}/2}\right) e^{\pm k_x^{line} x} \vec{u}_x \tag{5}$$

I_0 has the dimension Ampere and should be choosen to a proper value.

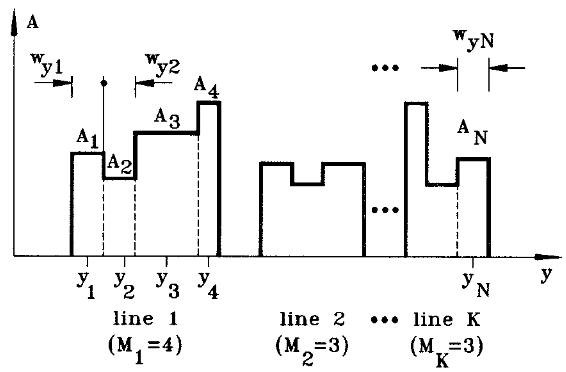

Fig.5 Discretisation of a coupled line system

Inserting the Fourier transform of eqn. (5) into the Fourier transform of eqn. (4) and using the method of moments we get the L wanted eigenvalues $k_x^{line(l)}$ as the zeros of

$$\det[Z] = 0 \tag{6a}$$

with

$$Z_{mn} = \int_{k_y} G_{xx}(k_x^{line}, k_y) \text{si}\frac{k_y w_{yn}}{2} \text{si}\frac{k_y w_{ym}}{2} e^{-jk_y(y_n - y_m)} dk_y \quad . \tag{6b}$$

The number of quasi TEM modes can be calculated with

$$L = 1, 2, ..., l, ..., K - 1 + K_{Ref} \qquad (7)$$

where K_{Ref} is the number of reflectors (1 or 2). The transverse current distribution on each of the K lines, which theirself are divided into M_k strips (see Fig. 5) is determined calculating the eigenvector $A^{(l)}$ of each eigenvalue l:

$$f_k^{(l)}(y) = \sum_{m=m_1}^{m_1+M_k-1} \frac{A_m^{(l)}}{w_{ym}} \mathrm{rect}\left(\frac{y-y_m}{w_{ym}/2}\right) \qquad (8)$$

with

$$k = 1, 2, ..., K \quad ,$$
$$l = 1, 2, ..., L \quad ,$$
$$m_1 = \sum_{\nu=0}^{k-1} M_\nu \quad \text{and} \quad M_0 = 1 \quad .$$

The propagation constant of the wave mode, the characteristic impedance of which has to be calculated can easily be identified with the knowledge of the transverse current distribution.

With this preinformation the circuit is simulated using spectral domain analysis again. Therefore the current on each semi-infinite line is discribed by outward travelling waves. Thus, *each infinite homogeneous line is built up by two semi-infinite lines which are connected by piecewise sinusoidal basis functions* $f_n(x)$. The equation for the current reads

$$\begin{aligned} \vec{J}_{tot}(x,y) &= \vec{J}_c + \vec{J}_{lines}^{semi} \qquad (9) \\ &= \sum_n^N I_n f_n(x,y)\vec{u}_x + \sum_k^K I_k^{out} f_k(x,y)\vec{u}_x \\ &= \sum_n^N I_n f_n(x) f_n(y)\vec{u}_x + \sum_k^K I_k^{out} f_k(y) e^{\pm k_x^{line} x}\vec{u}_x \quad . \end{aligned}$$

In this equation $f_{n,k}(y)$ are the transverse current distributions for the wave mode which has been chosen. The feeding of the model circuit is determined by the chosen wave mode and is simulated by M Δ-gap source(s) U_{Dm} ($m = 1, ..., M$) within one or more of the infinite homogeneous lines. Using the method of moments (modified Garlerkin's procedure) the longitudinal current distribution I_i is calculated solving the following set of linear eqations:

$$\sum_i^{N+K} I_i Z_{ji} = U_j \qquad \text{with } j = 1, ..., N+K \qquad (10)$$

and

$$Z_{ji} = \frac{1}{4\pi^2} \iint_{k_x k_y} G_{xx}(k_x,k_y) F_i(k_x,k_y) F_j^*(k_x,k_y) dk_x dk_y \quad ,$$

$$U_j = \begin{cases} U_{D\,m} & \text{for } m = j \\ 0 & \text{else} \end{cases}$$

In this formulation $F(k_x, k_y)$ is the Fourier transform of $f(x, y)$. After this step, the longitudinal as well as the transverse current distribution is known and the characteristic impedance of the wave mode can be calculated as has been discribed before.

EXAMPLES

The calculations of the characteristic impedances coincide exellently with those given in the literature: Fig. 6 shows the characteristic impedance of a single line embedded in multilayered media. Because of the different definition used in [1], there are small differences at higher frequencies between the results of [1] and those calculated with the new method.

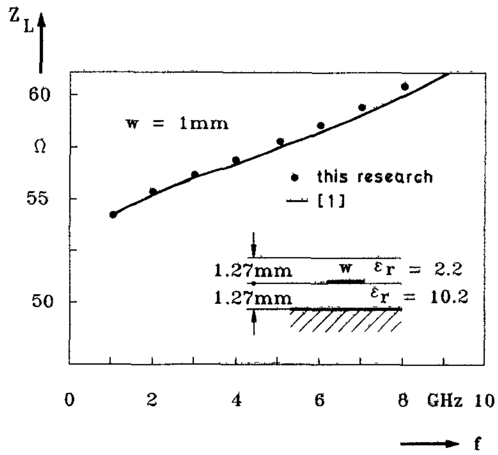

Fig. 6 Characteristic impedance of a microstrip line in layered media

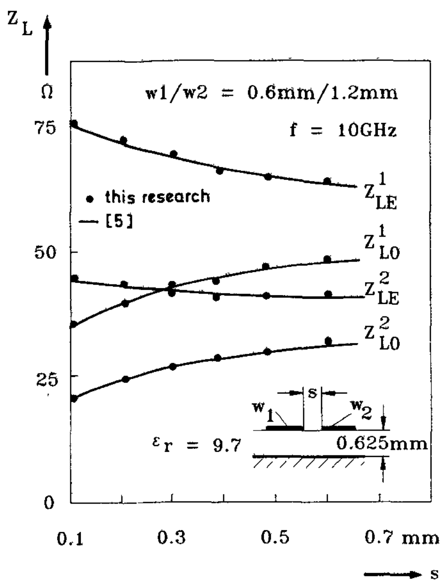

Fig. 7 Characteristic impedances of coupled lines

The even- and odd-mode paramenters for asymmetric coupled lines on a single layer are shown in Fig. 7 as a function of frequency. The lines are simulated as shown in Fig. 3. Again the values of the characteristic impedances agree very well with those given in the literature.

The next examples are for coupled lines in multilayered media, without a reflector. The first example is for a coplanar line (Fig. 8) which is covered by a superstrate, the permittivity of which increases from $\varepsilon_r = 1.0$ to $\varepsilon_r = 9.8$. Fig. 9 shows the effect of a small air region between the lines and the superstrate.

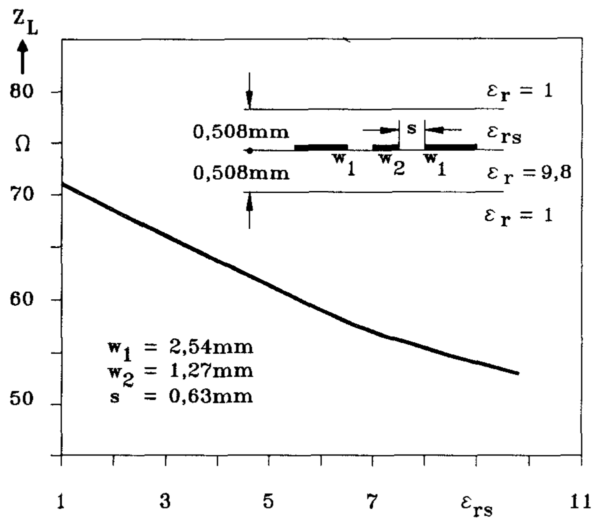

Fig. 8 Coplanar line with superstrate ε_{rs}

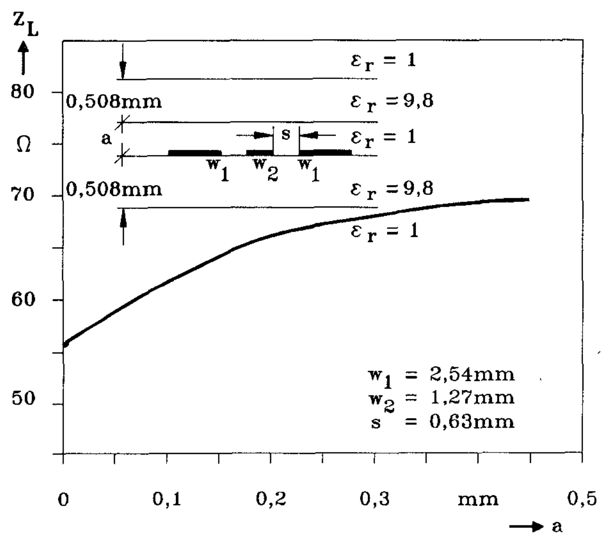

Fig. 9 Effect of an air region between substrate and superstrate

CONCLUSION

This paper describes a method by which the characteristic impedance of coupled lines in layered media can be calculated. By a small modification of the Green's function, this method can be applied to lines in different layers. Thus, this method can be used excellently for those planar TEM geometries for which no simple formulas are known.

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